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YIELDING SEISMIC RESPONSE OF CODE-DESIGNED SINGLE-STOREY ASYMMETRIC STRUCTURES

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SUMMARY

A study on the seismic performance of asymmetric structures with non-linear behaviour and random properties is presented. The structural response of single-storey models, designed using different code criteria is studied using both a deterministic and a probabilistic approach. The worst structural behaviour, in terms of ductility demands, is obtained for symmetric models considering uncertain properties. It is shown that an arbitrary increase of the total lateral strength of the structures does not lead to an increment of the structural safety proportional to the total lateral strength increase, implying that an expensive structure is not always the safest. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: asymmetric structures; seismic performance; random properties; non-linear behaviour; codes; Monte Carlo simulations

INTRODUCTION

A significant proportion of the observed damage in building structures during the 1985 Mexico earthquake can be attributed to ill torsional behaviour due to strength eccentricity.[1,2] In previous studies, it has been shown that stiffness and strength eccentricities modify the behaviour of structures under severe earthquake motions.[3–7] Frequently, these eccentricities are not evident for they are a consequence of the inherent variability in the values of the structural parameters that define the lateral response of the structures. Eccentricities due to random stiffnesses, strengths and centre of mass locations are present in most structures, even in those designed and built as nominally symmetric. Recent studies have evaluated the effect of several uncertainty sources on the torsional response of structures, most of them with linear elastic behaviour.[8–12] On the other hand, current regulations for seismic design of asymmetric structures have been defined based on studies of linear elastic structural models where non-linear effects are introduced through seismic performance factors.[13] However, the non-linear seismic behaviour of asymmetric structures differs from its linear elastic behaviour affected by seismic factors.[14] Since research studies in which the above considerations are jointly studied are not available, there is lack of information in this regard.

This paper is an effort to understand the basic characteristics of the seismic performance of code designed or nominal non-linear asymmetric structures with random parameters. The goal is to demonstrate the validity of code regulations or to justify if further research is needed for their modification.

This study is divided in two parts. In the first one, the problem is studied using a deterministic approach. The basis for the values of the coefficients involved in the design eccentricity in the current code for Mexico City, RDF87,[13] are reviewed. In the second part, a probabilistic study that evaluates the random response of

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structures with seismic torsional effects is presented. It is applied to shear building models with uncertainties in the location of the centre of mass, and with random stiffness and strength. The general procedure to carry out this study, shall be to perform a step-by-step non-linear dynamic analysis of the models in order to get their structural response.

The seismic performance of the structural models is evaluated in different ways. For the deterministic models, the ratio of the maximum ductility demand of asymmetric models to that of the corresponding symmetric structures is used. In the probabilistic case, a safety index for different values of failure ductility is determined.

From the results obtained, the importance of the in-plan distribution of structural elements strength is discussed. Finally, an alternative to the RDF87 design criterion, which can produce relatively safer structures, is proposed.

THEORETICAL CONSIDERATION

When the shear forces in the building storeys are not applied in the centre of torsion for each level, the translational response of the building is coupled with the rotational vibrations of its floor diaphragms. Such an effect is included in three-dimensional dynamic analyses of structures that consider the rotations and translations of the floors. As an alternative to the 3-D analysis, the RDF87 allows to carry a static analysis of the buildings taking into account only the interstorey translation. Torsional effects are included through seismic shear forces distributed among the resistant frames of the storey. The torsional moment in each storey is equal to the product of the shear force, times the corresponding design eccentricity, e_d, which causes the least favourable effect on each resistant element. Expressions for the design eccentricity in the RDF87 and in several building codes have the following format:

$$e_{d_1} = \alpha_1 e_s + \delta b \tag{1}$$

$$e_{d_2} = \alpha_2 e_s - \delta b \tag{2}$$

where e_s, is the distance between the centre of mass CM and the centre of torsion CT, α_1 and α_2 are coefficients that account for the differences between the static and dynamic analysis methods, and b is the larger dimension of the structure perpendicular to the direction of the earthquake excitation analysis. The accidental eccentricity factor, δ, takes into account possible ground rotations and the effect of variations of the structural properties and mass distributions in the building storeys.

Different values for the coefficients $\alpha_{1,2}$ and δ are recommended in various codes. Thus, for the Mexican code, RDF87,[13] $\alpha_1 = 1{\cdot}5$, $\alpha_2 = 1{\cdot}0$ and $\delta = 0{\cdot}1$; for the U.S. code, ATC,[15] $\alpha_1 = 1{\cdot}0$, $\alpha_2 = 0$ and $\delta = 0{\cdot}05$; for the Canadian code, NBCC,[16] $\alpha_1 = 1{\cdot}5$, $\alpha_2 = 0{\cdot}5$ and $\delta = 0{\cdot}1$; and for the European code, CEB,[17] considering the design eccentricity measured from the CT as in the other codes, $\alpha_1 = 1{\cdot}5$, $\alpha_2 = 1{\cdot}0$ and $\delta = 0{\cdot}05$.

From the above values, it can be noticed that some codes such as the RDF87 and the NBCC, subtract the negative torsion shear from the direct shear. Other codes, such as the ATC, consider only the increment in that shear, neglecting the negative shear due to torsion.

The application of the torsional design criteria provides the minimum value for the strength of the resisting elements.[13] Therefore, the structural damage distribution among elements will be a function of the coefficients α_1, α_2 and δ.

STRUCTURAL MODEL AND COMPUTATIONAL TOOLS

In this paper, a one-storey shear building designed to reproduce the basic response characteristics observed in real structures is considered. The structural model has three non-linear resistant elements fixed to their base, and rigidly connected to the floor system (Figure 1). The CM is located in the centre of gravity of the building plan. The model was designed using the seismic torsion design criteria described before. The stiffness

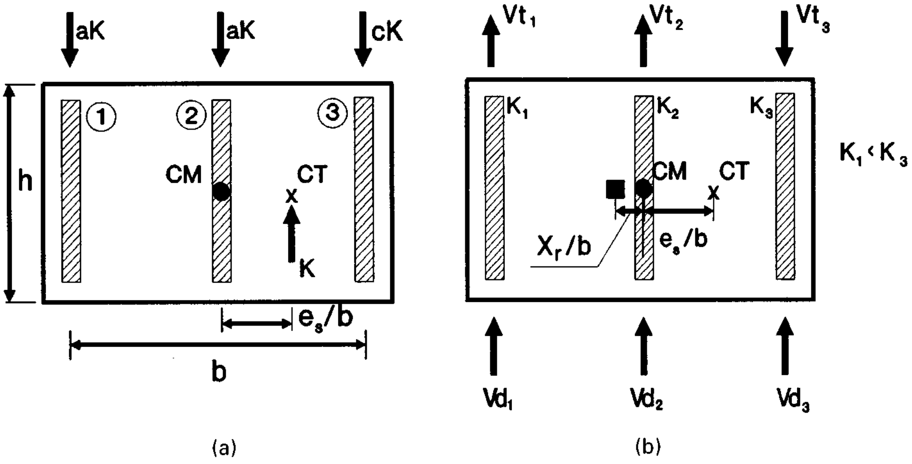


Figure 1. Structural model: (a) stiffness elements assignment and (b) earthquake forces computation

of the structural elements is assigned increasing the stiffness of the border element, keeping constant the total lateral stiffness of the system, as indicated in Figure 1(a). From this figure, using statics, if $b = 1$:

$$\sum M = Ke_s + \tfrac{1}{2}aK - \tfrac{1}{2}cK = 0$$

$$2a + c = 1$$

then

$$c = 1 - 2a$$

$$a = \tfrac{1}{3} - \tfrac{2}{3}e_s$$

Earthquake forces were computed with the equivalent lateral force method using the design spectrum of the RDF87[13] which specifies a seismic coefficient as a function of the fundamental period T of the structure and a seismic performance factor Q. The seismic shear that the ith structural element must resist is indicated in Figure 1(b). It is determined as the algebraic sum of the element direct shear V_{d_i} (proportional to its stiffness) and the torsion shear V_{t_i}, computed using equations (1) or (2) depending on which one causes the least favourable effect on the resisting elements.

A non-linear elastoplastic model is used to characterize the structural restoring force. The effects of viscous damping and deterioration of the structural properties were not considered.

The dynamic analyses of the structural models were performed using the computer program DYNDIR,[18] which allows one to perform non-linear analyses of three-dimensional structures.

STRUCTURAL PARAMETERS

From the results in previous research studies,[3–8] it may be concluded that, in order to characterize the seismic behaviour of non-linear asymmetric buildings, the structural models require more parameters than those used for elastic models such as the strength of the structural elements.

In this paper, the parameters investigated are: four natural vibration periods $T = 0{\cdot}5$, $1{\cdot}0$, $1{\cdot}5$ and $2{\cdot}5$ s; four values of the normalized structural eccentricity $e_s/b = 0{\cdot}0$, $0{\cdot}1$, $0{\cdot}2$ and $0{\cdot}3$; three values for the aspect ratio of the plan building $h/b = 0{\cdot}5$, $1{\cdot}0$ and $2{\cdot}0$, and their corresponding values of the lateral to torsional frequency ratio; and two values for the seismic performance factor[13] $Q = 2$ and 4. The positive sign in the e_s/b values indicates that the CT is located to the right-hand side of the CM and vice versa (Figure 1).

Frequent observations of structural damage caused by earthquakes[20,21] have shown that the real strength R_r of a structure differs from, and normally exceeds, the calculated or nominal strength R_n. This difference may be attributed to variations in material strength and geometry of the structural members, as well as inherent variabilities in the parameters of the equations used to compute the element strength, among other factors.[21] Such a difference, usually on the safe side, is defined by an over-strength factor (OSF $= R_r/R_n$) always greater than unity. For scale models, recent investigations have found that the computed values of OSF could be as high as 7·0.[22] In order to represent the real strength of the models in a more rational way, four gross values for the OSF $= 1{\cdot}0$, 1·5, 2·0 and 3·0, were considered. The seismic performance factor Q used to design the structural models accounts, in some way, for the reserves of capacity of the structures to resist lateral loads,[23] and hence it could be related to the OSF. Nevertheless, there is no explicit information quantifying the capacity reserve included in the seismic performance factor[24] Q. As the real strength of a structure is usually not known, the above four values for the OSF were adopted.

Although the RDF87 Code specifies that the negative shear due to torsion be subtracted from the direct torsion, in Mexico City's engineering practice the strength of the structural elements is obtained subtracting only a percentage of the negative shear.[25] Therefore, the position of the storey-resistant force in real structures could have a location different from the nominal value computed from equations (1) and (2). In this paper, in order to study the effect of strength distribution on the structural response, seven in-plan distributions of element strengths that produce cases with different locations of the resistant force will be studied in addition to the cases where the storey resistant force is located at its nominal position. The later are studied using the normalized parameter X_r/b, which defines the distance between the storey-resistant-force location and the centre of mass and can take positive and negative values to the right and left of the CM, respectively.

Here, the nominal position of the storey-resistant forces is the one that corresponds to the application of the torsion design criteria specified by the codes. Values for X_r/b are obtained increasing the resistance of the structural element on the side towards the storey-resistant force is to be located. In this way, the design regulations still apply since the resistance of each of the structural elements is never lower than the nominal values obtained from the torsion design criterion.

STATISTICAL ANALYSIS OF STRUCTURAL PROPERTIES

As is the case in other processes in nature, the information related with loads and the structural properties of the building structures is not free from uncertainties. Nevertheless, it is possible to use it through probabilistic models which based on statistical properties, are used to determine the probability distributions of the required structural parameters. In this paper the centre of mass location, the stiffness and the strength of the structural elements, are modelled as random variables. The probability distributions for the stiffness and strength of the structural elements are log-normal, with coefficient of variation of 20 per cent.[19] The mean value of the structural elements' strength is obtained as its nominal value multiplied by the over-strength factor OSF. For the centre of mass position, the probability distribution function was considered Gaussian, with statistical parameters obtained considering the $\pm b$ range specified by the RDF87.[13]

Additionally, a correlation coefficient $\rho = 0{\cdot}0$ and 0·5 for the stiffness and the strength of each resistant element was used. This consideration allows one to assess the effect of statistical correlation of the structural properties on the seismic performance of the building models.

EARTHQUAKE EXCITATION

To study the behaviour of structural models located on soft soil in the Valley of Mexico, the E–W component of the SCT motion recorded during the 1985 Michoacan earthquake was used as input. In order to reproduce the effect of a seismic environment, a family of records simulated from the Michoacan earthquake was obtained.[26] These records were used for the analysis of a non-linear asymmetric structural model; it was

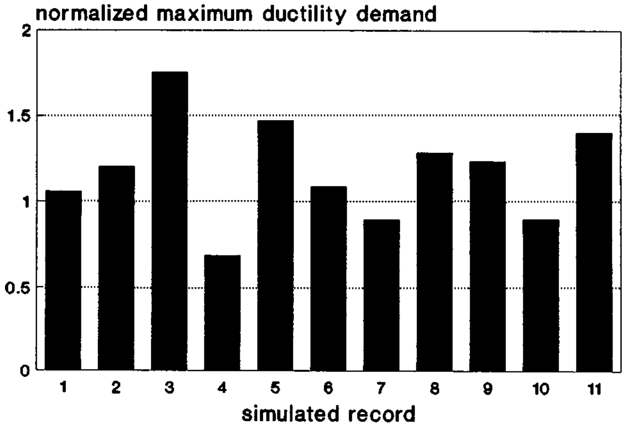


Figure 2. Maximum structural response of an inelastic asymmetric structural model

found that some of them did not produce a non-linear behaviour in the structural elements, as can be observed in Figure 2 for the maximum ductility demand of an asymmetric model. Hence, it was necessary to scale the records.[19] Four scale criteria were evaluated based on the use of (1) the peak ground acceleration, (2) the maximum spectral pseudo-acceleration, (3) the Arias intensity,[27] and (4) the total energy earthquake inputs.[28]

The scaled simulated records were used for non-linear analyses of an asymmetric structural model with random properties. The relative error of the statistical parameters (mean and coefficient of variation) for the maximum ductility demand was obtained. This error is relative to the same statistical parameters computed for the non-deterministic model excited only by the SCT-EW record. The results show that the total energy scale criterion always produces the minimum relative error (Figure 3), and that it is less than 1 per cent (Figure 3(a)). Thus, the statistical properties of the random response of the structural model excited by the family of simulated records have almost the same values as the ones obtained using only the SCT-EW record. Therefore, in the present study, this record will be used alone.

DETERMINISTIC STUDIES

Figure 4 shows the effect of different locations of the storey-resistant force X_r/b, on the variation of the lateral total strength S_a. The strength S_a of the asymmetric models, is normalized with respect to the symmetric model strength S_s obtained for structural models designed according to the RDF87 torsion design criterion. In Figure 4, each curve is associated with a different value of e_s/b. The dots into the curves indicate the nominal position of the storey-resistant force. It may be seen that the total lateral strength variation depends on the relative position of the centre of torsion and on the position of the storey-resistant force. It may also be observed that if the e_s/b values are positive, then X_r/b has negative values, which suggests that the torsion design criteria (equations (1) and (2)) produce a relation between the nominal stiffness and the nominal strength of the structural elements. This relation could be very important, especially when the structure undergoes non-linear behaviour, given that the strength distribution will control the damage distribution over the structural elements.

Figure 5 shows the effect of the static eccentricity of a model designed with the torsion design criteria described above (horizontal axis), over the stiffness and the strength of the structural elements. The stiffness and the strength of element 1 (the weaker element) of the structural model, divided by the corresponding stiffness and strength of element 3 (the stiffer element), is shown in the vertical axis of Figure 5 for different e_s/b values. For some torsion design criteria, as the ratio of the stiffness of the elements decreases, their

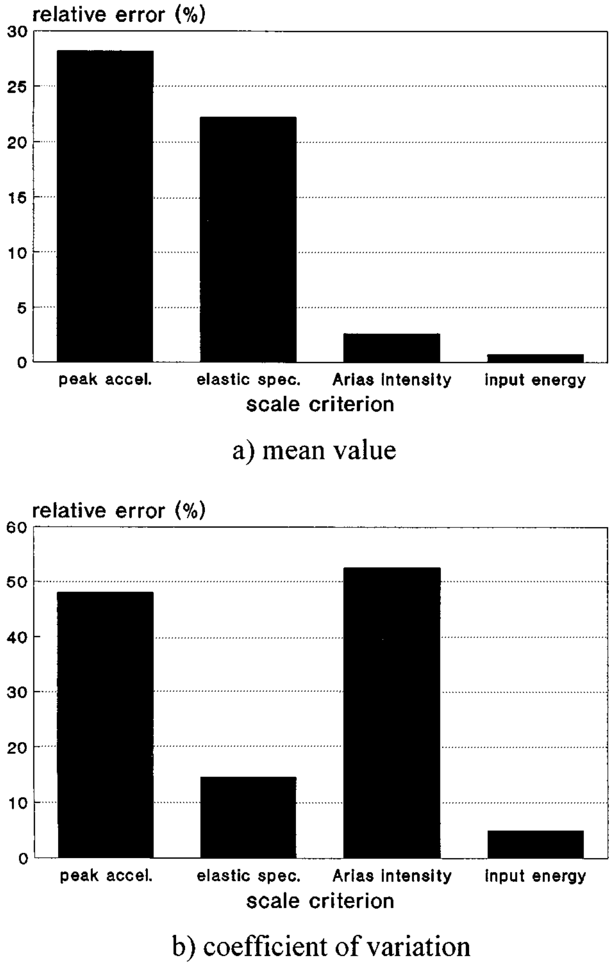


Figure 3. Relative error of the statistical parameters of the maximum ductility demand of a non-deterministic asymmetric structural model: (a) mean value and (b) coefficient of variation

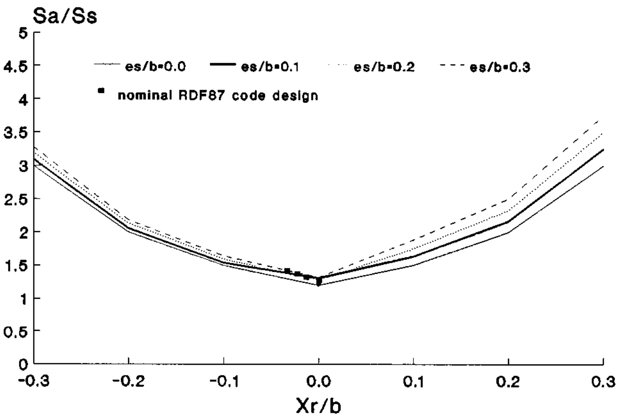


Figure 4. Effect of different locations of the storey-resistant force, over the normalized lateral total strength (RDF87 design criterion)

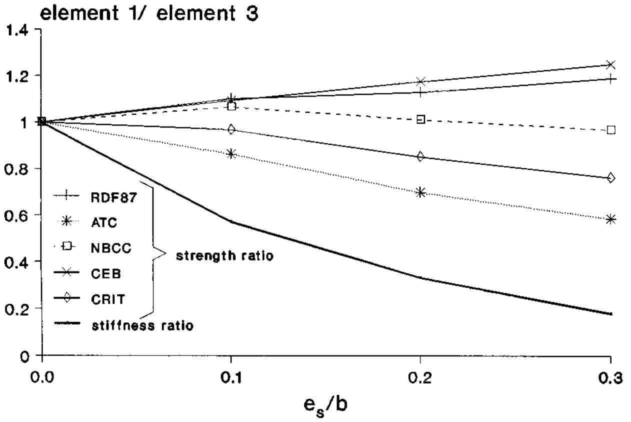


Figure 5. Ratio of the stiffness and strength of the structural elements of models designed for torsion

strength ratio increases, implying that the nominal stiffness of the structural elements is not proportional to its nominal strength. Thus, apparently, this disagreement between the stiffness and strength of the resistant elements is produced through the values of the coefficients α_1, α_2 and δ.

Figure 6 shows the relationship among the aspect ratio h/b of the plan building, the lateral to torsional frequency ratio, and the e_s/b values for the studied structural models. Figure 7 shows the effect of different values of e_s/b and h/b (and indirectly the frequencies ratio), on the variation of the maximum ductility demand D_a of the asymmetric models, normalized with respect to the symmetric model maximum ductility demand D_s. The models are designed according to the RDF87. In Figure 7, each curve is associated with a different value of h/b and the seismic performance factor Q. The dashed lines of the plot correspond to structural models designed with $Q = 2$, and the solid lines with $Q = 4$. It may be seen that using the two different seismic performance factors in the design, two different structural models are obtained; in spite of having the same lateral to torsional frequency ratio as can be seen in Figure 6, both models show very different structural behaviour. This was also observed in a previous study,[29] where for three values of the plan building aspect ratio studied, $h/b = 1{\cdot}0$ shows a minimum variation, even for models designed using $Q = 2$ and 4 (Figure 7). Thus, in order to get a non-extreme structural response, the aspect ratio $h/b = 1{\cdot}0$ of the plan building is considered for the non-deterministic structural models.

The ratio of the maximum ductility demand of the structural elements D_a, normalized with respect to the reference symmetric model D_s, is plotted in Figure 8 as a function of the storey-resistant-force position X_r/b. The models were designed using the RDF87 with $T = 1{\cdot}0$ s, $Q = 4$, OSF $= 1{\cdot}5$, and $e_s/b = 0{\cdot}0$, $0{\cdot}1$, $0{\cdot}2$ and $0{\cdot}3$. The vertical line in each plot in Figure 8 indicates the nominal position of the storey-resistant force. It can be observed that the nominal design is located in a zone where the maximum ductility demand could be increased if, due to some variation of the structural strength, the CT and the position of the storey-resistant force lie in opposite sides with respect to the CM.

Using trial-and-error tests, and a heuristic procedure,[19,29,30] alternative values for the coefficients α_1 and α_2 of the design eccentricity equations (1) and (2), were found. In this work, such design criterion is named CRIT and the coefficients are taken as $\alpha_1 = 1{\cdot}0$ and $\alpha_2 = 0{\cdot}5$ and $\delta = 0{\cdot}1$.

Figures 5, 9 and 10 show results for the CRIT design criterion. In Figure 5, it may be observed that the stiffness ratio and the strength ratio for the structural elements, do not differ in the CRIT as they do in the RDF87 criterion, which implies a relatively greater proportionality between the nominal stiffness and the nominal strength of the structural elements. Figure 9 shows that CRIT presents a better trend (observed through the signs), between the parameters e_s/b and X_r/b, yielding a higher increase in the S_a/S_s ratio, especially when these parameters have different signs. Figure 10 shows that the maximum ductility demand

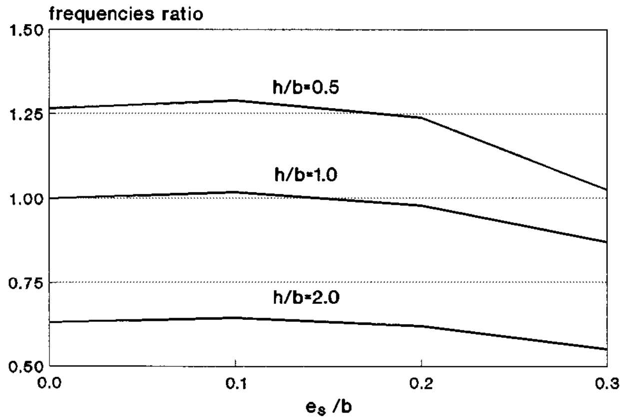


Figure 6. Relationship among the aspect ratio h/b of the plan building, the lateral to torsional frequency ratio, and the e_s/b values for the studied structural models

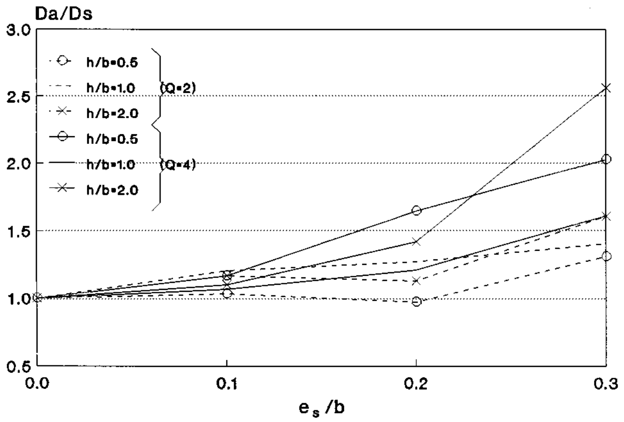


Figure 7. Effect of e_s/b and h/b (and indirectly the frequencies ratio), on the normalized maximum ductility demand D_a/D_s

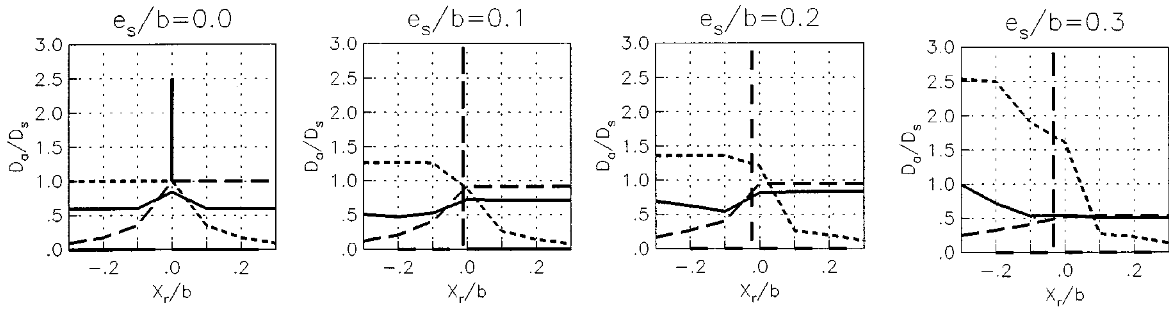


Figure 8. Effect of the position of the storey-resistant force over the normalized maximum ductility demand of the structural elements, RDF87 design criterion, $T = 1{\cdot}0$ s, $Q = 4$, OSF $= 1{\cdot}5$. ——— element 1, – – – – element 2, · · · · element 3

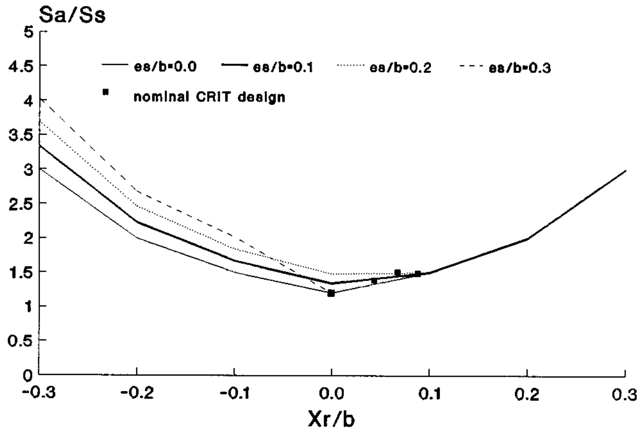


Figure 9. Effect of different locations of the storey-resistant force over the normalized lateral total strength (CRIT design criterion)

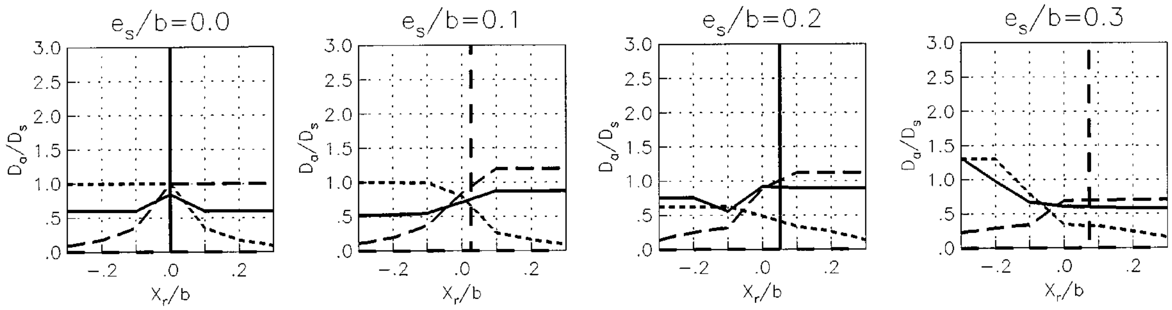


Figure 10. Effect of the position of the storey-resistant force over the normalized maximum ductility demand of the structural elements, CRIT design criterion, $T = 1{\cdot}0$ s, $Q = 4$, OSF $= 1{\cdot}5$. ——— element 1, –––– element 2, · · · · element 3

ratio, D_a/D_s, of the structural elements designed with the CRIT criterion, is lower than that generated by RDF87 (Figure 8). The shape of the maximum ductility demand curves are smoother than those of RDF87, which implies a 'better' damage distribution among the structural elements. For the nominal designs, the position of the storey-resistant force (vertical line), the values of D_a/D_s are equal to or less than unity, which means that, from a deterministic point of view, the behaviour of the asymmetric model is similar to the reference symmetric model.

Results for structural models designed with RDF87 and CRIT criteria for $T = 0{\cdot}5$ and $1{\cdot}5$ s, $Q = 2$ and 4, and $b/h = 0{\cdot}5$ and $2{\cdot}0$, show a similar trend to the one presented above.[19]

MONTE CARLO SIMULATIONS

Due to the large computational effort, it was necessary to look for some strategy in order to perform the minimum number of simulations without loosing precision in the statistical analysis of the results. In the literature, several works with this objective were found. The Kolmogorov–Smirnov test was used,[31] taking as absolute error the maximum value of the difference between the theoretical and the simulated probability distribution. Computing the probability of occurrence of this error, the minimum number of simulations for which the minimum value of the absolute error has some confidence level was established. The concept of Barnard's test[32] was used to compute the size of the set of simulated samples by the level of significance at which testing is to be carried out, and the procedure adopted for judging significance.[33,34] In this way, the

number of Monte Carlo trials required to achieve a particular level of reliability can be determined. However, the minimum number of Monte Carlo simulations using these concepts is still very high. Because of this, strategies such as the Latin Hypercube Sampling, Stratified Sampling, Two Point Estimations in Probability[35] and Multi-point Estimations in Probability,[36] were evaluated.[30] It was found that using the latter, the results from the statistical analysis of the outputs were excellent compared with those of a large random sampling commonly used in the Monte Carlo method. Thus, for the simulation of the structural properties of the models, the Monte Carlo method was used. The structural properties were simulated according to the Multi-point Estimations method using three probability concentrations for the stiffness and the strength, and two for the centre of mass location.

PROBABILISTIC MODELS' RESULTS

Due to inherent variabilities in the parameters that define the structural elements, the behaviour of any structure becomes a complex phenomenon. In order to evaluate the non-linear random structural response of asymmetric buildings, the concept of maximum ductility demand, widely utilized by earthquake engineers, is used.

The structural safety of the models designed with the RDF87 and CRIT criteria, is evaluated applying structural reliability theory through the reliability index ß. The objective of the ß index is to asses the safety level of a structure for a defined limit state. Thus, the reliability index is a measure of the adequacy of a structural design. Relating the random structure capacity, C, and the random demand, D, through a safety margin designated as $S = C - D$, the reliability index is defined as the number of times the standard deviation of the safety margin can be contained into its mean value.[37] In this study, the safety margin is evaluated as the difference between the ductility of failure DF, and the maximum ductility demanded by the structure $D_{\max}$, as follows:[38]

$$Z = \mathrm{DF} - D_{\max}$$

So, the reliability index ß is defined as

$$\beta = \frac{\bar{Z}}{\sigma_z}$$

where

$$\bar{Z} = \ = \mathrm{DF} - \bar{D}_{\max}$$

$$\sigma_Z^2 = \sigma_{\mathrm{DF}}^2 + \sigma_{D_{\max}}^2 - 2\mathrm{cov}[\mathrm{DF}, D_{\max}] = \sigma_{D_{\max}}^2$$

In these equations, $D_{\max}$, $\sigma_{D_{\max}}^2$ are the mean value and variance of the maximum ductility demand, respectively, σ_{DF}^2 is the variance of the ductility of failure, and $\mathrm{cov}[\mathrm{DF}, D_{\max}]$ is the covariance between DF and $D_{\max}$.

Given that the true value of the maximum ductility for which a structure fails is unknown, three deterministic values for the ductility of failure are used in this work, DF = 4, 6 and 8.[38] The statistical properties of the maximum ductility demand are obtained from the Monte Carlo simulations.

As the capacity and the demand of a structure are random variables, the safety margin will be also random. If the capacity and the demand have normal probability density functions, the probability of failure P_f of a structure can be computed as[37]

$$P_f = \varphi(\beta)$$

where, $\varphi(\ )$ is the standard normal distribution function. Thus, for ß = 0, $P_f = 0{\cdot}5$; for ß = 1, $P_f = 0{\cdot}159$; and for ß = 10, $P_f = 2{\cdot}55 \times 10^{-10}$.

Figure 11 shows the reliability index ß for the structural models designed with the RDF87 and CRIT criteria. Different values for the normalized structural eccentricity e_s/b were used. The plots correspond to

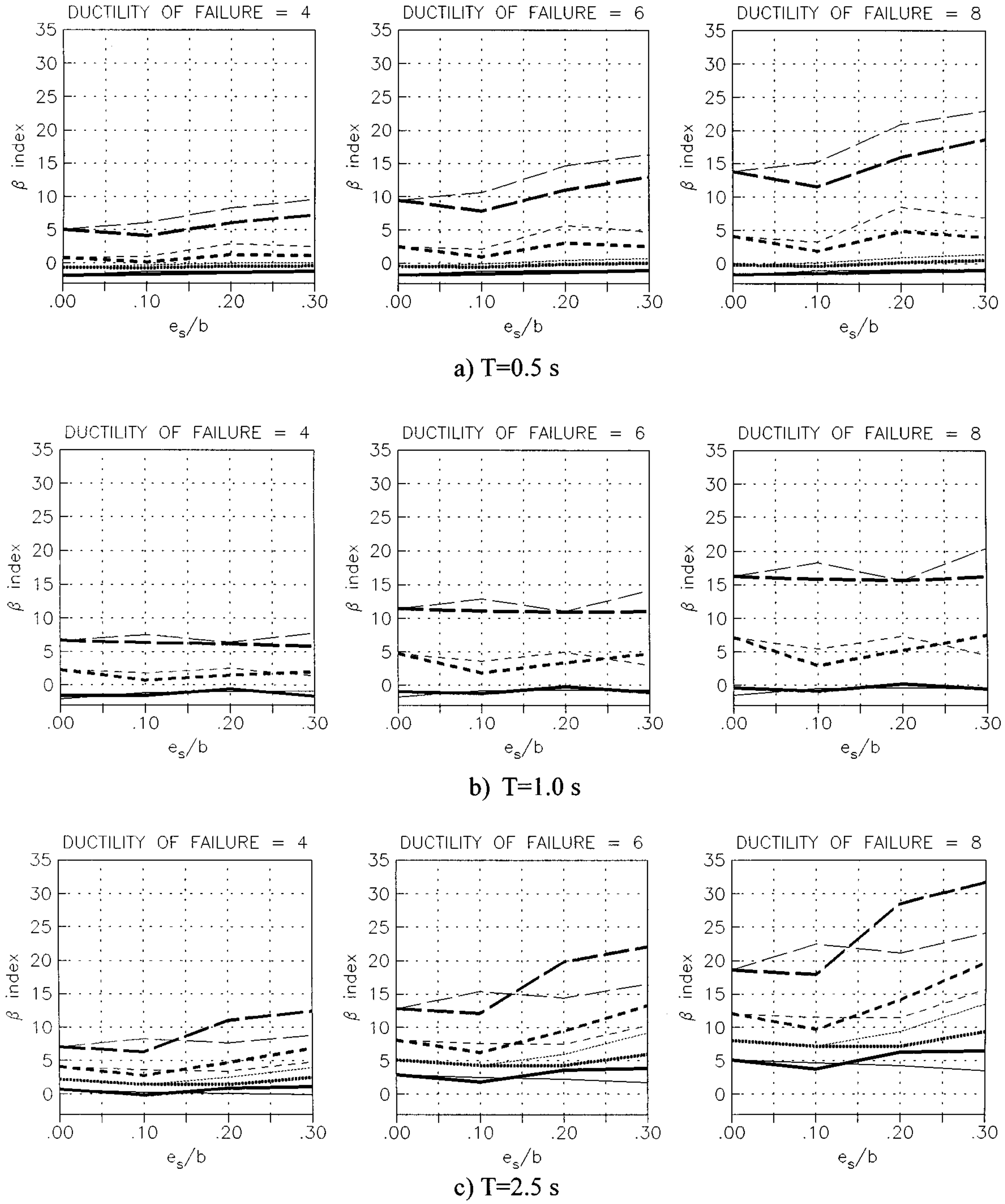


Figure 11. Reliability index for the structural models designed with the RDF87 (thin lines), and the CRIT (thick lines) criteria, $Q = 4$. ——— OSF = 1·0, · · · · OSF = 1·5, – – – – OSF = 2·0, — — — OSF = 3·0 for T values of (a) 0·5 s, (b) 1 s and (c) 2·5 s

models with fundamental vibration periods $T = 0{\cdot}5$, $1{\cdot}0$ and $2{\cdot}5$ s, and structural over-strength factors OSF $= 1{\cdot}0, 1{\cdot}5, 2{\cdot}0$ and $3{\cdot}0$. All the models were designed using $Q = 4$. For both criteria it can be seen that the reliability index values are very similar, and that ß increases with the over-strength factor and with the fundamental vibration period. In general, the variation of the ß values is higher for the models with ductility of failure DF $= 8$. For the CRIT designs, the most affected structural models for the variation of the static eccentricity are those with $T = 2{\cdot}5$ s.

In Figures 12–14 the variation of the ß index due to the e_s/b and X_r/b values is shown. In these cases OSF $= 1{\cdot}5$. Four values for the normalized static eccentricity and three for the ductility of failure were used. In these figures the dots into the curves indicate the nominal designs. For the RDF87 designs it can be observed that for structural models with values of e_s/b and X_r/b on the same side with respect to the center of mass, the reliability index ß increases until a limiting value. Vice versa, if e_s/b and X_r/b differ in signs, the reliability index decreases. The latter can be attributed to the lack of consistency between stiffness and strength of the structural elements, and to the fact that when all of the structural elements are yielding, the CT changes its location towards the nominal X_r/b. As much as this new point is far away from e_s/b (the original CT position), a greater additional torsional effect could be produced, affecting the whole structural

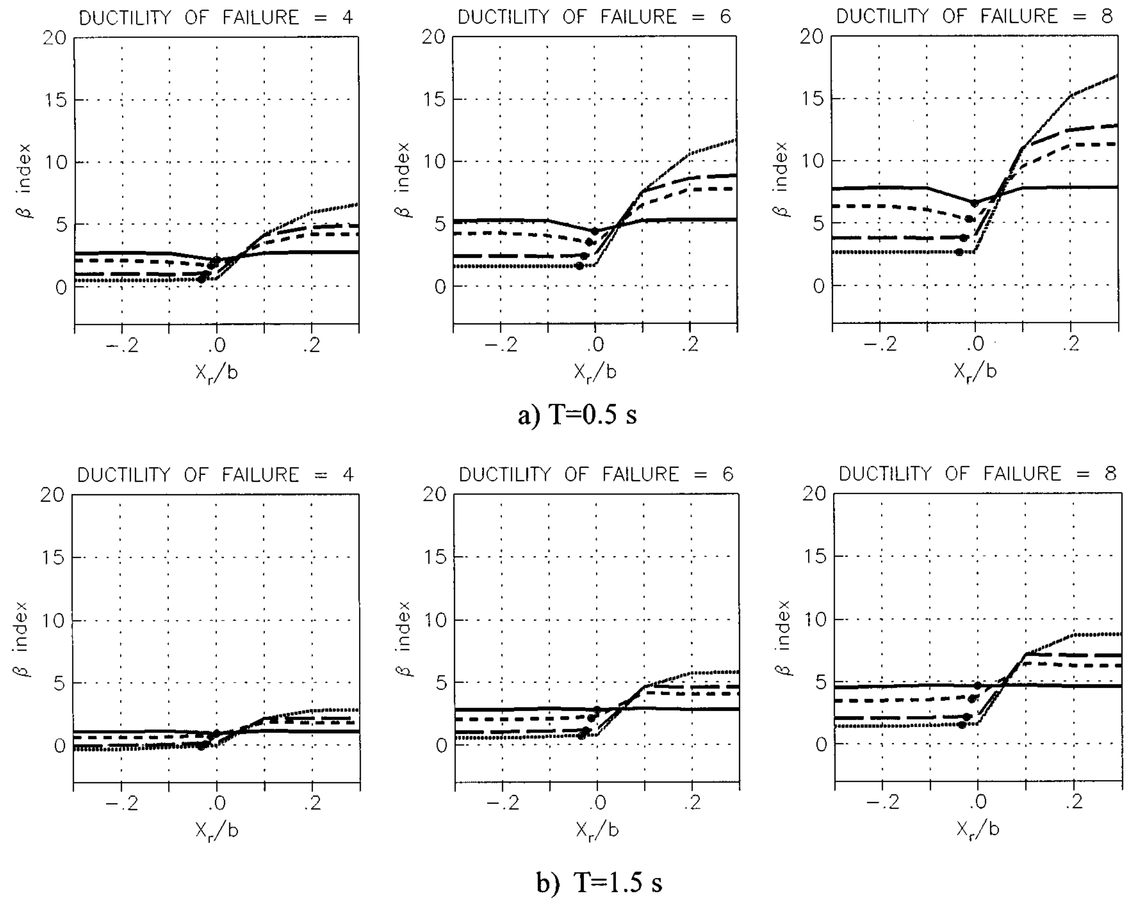
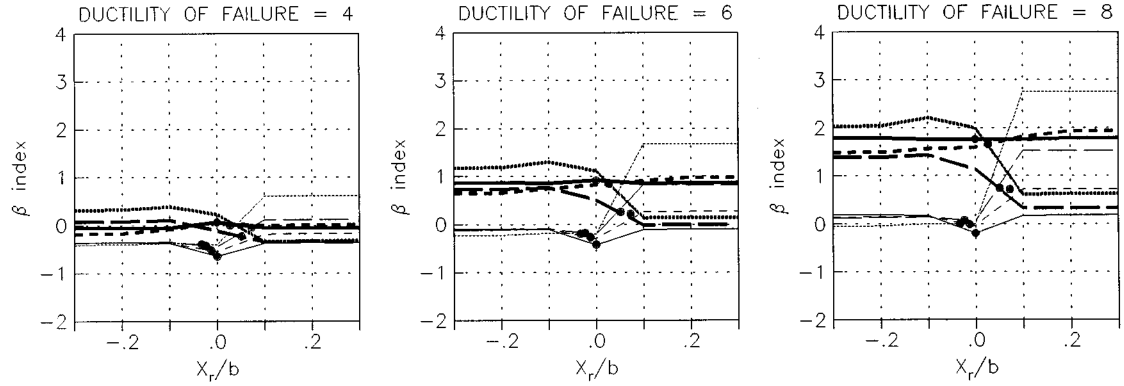
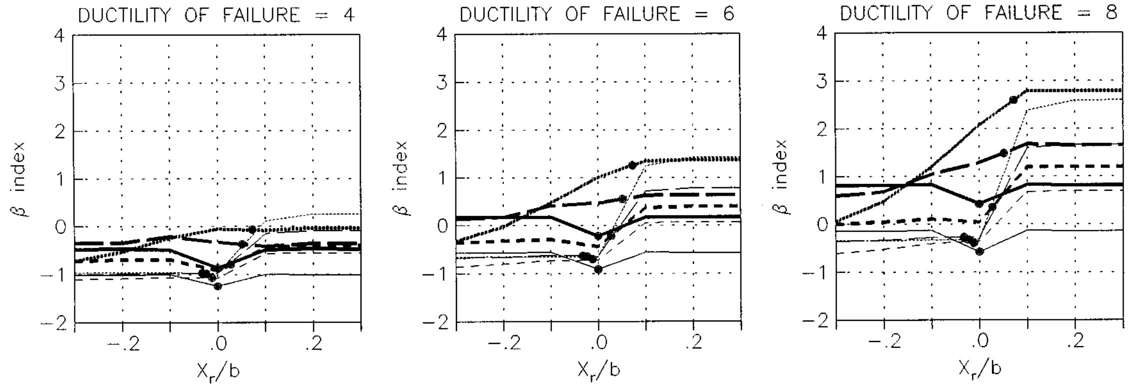


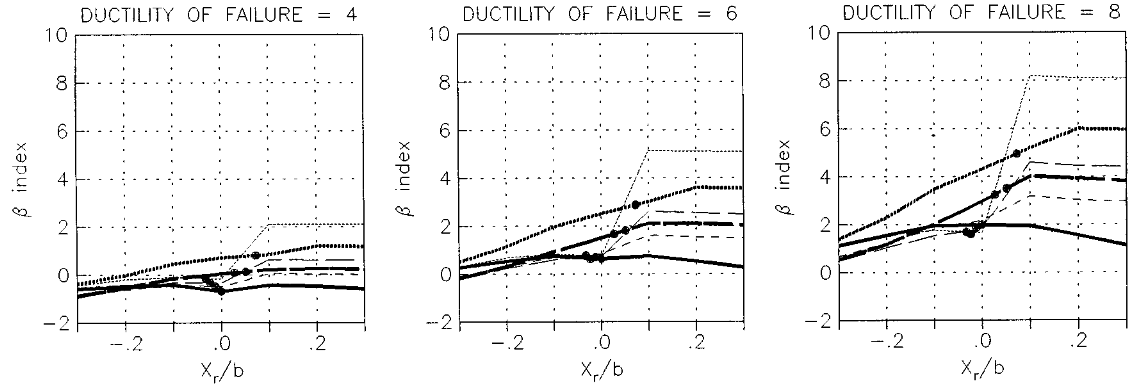
Figure 12. Variation of the reliability index due to the static eccentricity and the position of the storey-resistant force. RDF87 design criterion, $Q = 2$, OSF $= 1{\cdot}5$, no correlation between the stiffness and the strength of the structural elements. ——— $e_s/b = 0{\cdot}0$, ---- $e_s/b = 0{\cdot}1$, —— $e_s/b = 0{\cdot}2$, $\cdot\cdot\cdot\cdot$ $e_s/b = 0{\cdot}3$. (● nominal design) for T values of (a) 0·5 s and 1·5 s



a) T=0.5 s



b) T=1.0 s



c) T=1.5 s

Figure 13. Variation of the reliability index due to the static eccentricity and the position of the storey-resistant force. RDF87 design (thin lines), CRIT design (thick lines) criteria, $Q = 4$, OSF = 1·5, no correlation between the stiffness and the strength of the structural elements. ——— $e_s/b = 0{\cdot}0$, – – – $e_s/b = 0{\cdot}1$, —— $e_s/b = 0{\cdot}2$, · · · · $e_s/b = 0{\cdot}3$ (● nominal design) for T values of (a) 0·5 s, (b) 1·0 s and (c) 1·5 s

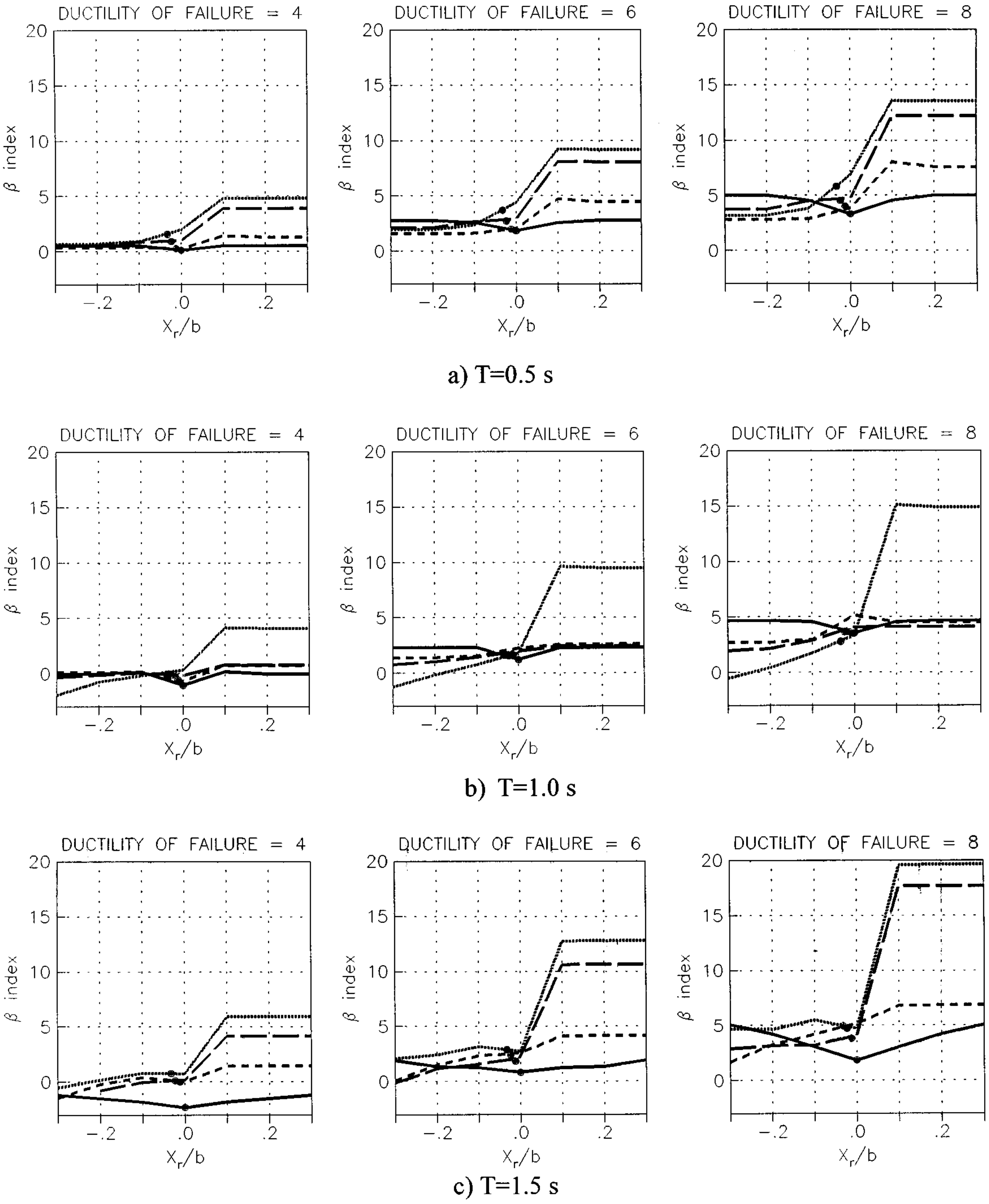


Figure 14. Variation of the reliability index due to the static eccentricity and the position of the storey-resistant force. RDF87 design criterion, $Q = 4$, OSF $= 1{\cdot}5$, correlation coefficient $\rho = 0{\cdot}5$ between the stiffness and the strength of the structural elements. ——— $e_s/b = 0{\cdot}0$, — — — $e_s/b = 0{\cdot}1$, – – – $e_s/b = 0{\cdot}2$, · · · · $e_s/b = 0{\cdot}3$ (● nominal design) for T values of (a) 0·5 s, (b) 1·0 s and (c) 1·5 s

behaviour. As can be seen in Figures 12–14, for the RDF87 criterion this effect increases the ß index value for asymmetric structures with $e_s/b = 0{\cdot}1$. An apparent reason for such behaviour is that in order to generate a particular X_r/b value, as was explained before, the strength of one of the border elements (element 1 or 3), is increased. If this is the weak element (element 1), then it turns into the strongest element, and the maximum ductility is reached by element 3 (Figure 8), no matter which is the stiffer element. Vice versa, if the strength of the stiff element is increased (element 3), then it turns into the stiffer and the stronger element, and a consistency between stiffness and strength of the element, greater than that originally generated in the nominal design, is produced. In this case, the maximum ductility demand decreases and the reliability index increases as can be seen in Figures 8 and 13, respectively.

In addition to the effect of the strength increase explained above for the stiffer element, the change of the ß index around $X_r/b = 0{\cdot}1$ can be attributed to the rise of the global structural strength of the asymmetric models.

In Figure 12 it can be observed that for models designed using $Q = 2$, the reliability index decreases as the fundamental vibration period increases. In this figure only two extreme cases of the fundamental vibration period are presented ($T = 0{\cdot}5$ and $1{\cdot}5$ s). Results for the model with $T = 1{\cdot}0$ s are proportional to the above. For models designed using $Q = 4$ (Figure 13), the trend of the ß index for the RDF87 designs is similar to the models with $Q = 2$ (Figure 12), but now the ß index increases with the fundamental period. Comparing Figures 12 and 13, one can see that, as it could be expected, structural reliability increases as the seismic performance factor decreases.

In Figure 14 the results of the ß index for models designed with RDF87 are shown. In these cases $Q = 4$ and the value for the correlation coefficient between the stiffness and the strength of each element is $\rho = 0{\cdot}5$. In this study, the correlation coefficient is the measure of the linear interdependency between random stiffness and random strength. As it is related with the covariance of both random variables , if it is large and positive, their values tend to be both large or both small, relative to their respective means. In this case, given that the correlation coefficient is positive, the values for the ß index are greater than those obtained for the $\rho = 0{\cdot}0$ cases (Figure 13), as it could be expected. Again, in these figures, it can be observed that if X_r/b presents the same sign as the normalized static eccentricity e_s/b, the reliability index value increases and vice versa. This increase or decrease is towards a limiting value. After this value is reached, there is no significant change in the value of ß. In these cases the nominal designs (dots into the curves), are located in a low-value area of the structural-reliability index. Comparing Figures 13 and 14 for the RDF87 designs, it is shown that the statistical correlation between the stiffness and strength of the resisting elements increases the values of the reliability index ß.

In Figure 13 the values of the reliability index for the models designed with the CRIT criteria for $Q = 4$ are shown. In these models there is no correlation between stiffness and strength. Reliability increases with the fundamental vibration period, and the ß index is slightly affected by the value of the ductility of failure. In these cases the nominal designs, yield values of the ß index greater than those observed for the RDF87 nominal designs.

From Figure 13, it may be observed that the values for the ß index obtained from the models designed using the RDF87 criterion, present a greater variation than the CRIT designs.

CONCLUSIONS

In this paper, the importance of considering the non-linear behaviour, the random stiffness and strength of the structural elements, and the uncertain location of the centre of mass, on the performance of asymmetric building structures was studied. Analyses were performed using the SCT record in Mexico City from the Michoacan earthquake of 1985; the results obtained may not hold for asymmetric buildings on soils with different characteristics from those of the soft soils from the SCT site. This work was based on a probabilistic method that provides the assessment of the reliability of the structures. For the structural models studied, the strength distribution of the structural elements must be modified in order to increase the structural safety.

For the deterministic models considered, the CRIT design criterion, presented in this paper, produces a theoretical performance better than that of the RDF87 criterion. In both criteria, the nominal designs increase the lateral total strength of the models in the same proportion.

The correlation between stiffness and strength of the elements, and the structural over-strength, increases the reliability index ß. The increase of the seismic performance factor Q, decreases the values of the ß index.

The lowest values for the structural-reliability index were found for the symmetric models in both design criteria (RDF87 and CRIT). One reason for this could be the increase of the strength for the non-symmetric models due to the torsion design. Therefore, all the structures, even those considered as symmetric, must be designed including some torsional effects.

The increase of lateral strength in the structural models did not yield a proportional increase in the structural reliability. From the results obtained it may be observed that the structural safety grows, if the static eccentricity and the position of the storey-resistant force are on the same side with respect to the centre of mass. On the other hand, when the static eccentricity and the position of the storey-resistant force, are in opposite positions with respect to the centre of mass, structural safety decreases. In both cases, the total lateral strength of the structure was increased, so the non-rational increase of the structural strength in the non-symmetric structures, does not produce safer structures; in the best case only the cost of the structures will be increased. The increase or decrease of the structural-reliability index is towards a boundary value. Due to this fact, in order to get safer asymmetric structures, the variation of the position of the storey-resistant force, due to the application of a seismic torsion design criterion, should be inside a limited space, beyond which the structural safety variations are insignificant. This goal could be reached if the position of the storey-resistant force moved towards a location between the centre of mass and the centre of torsion.

The conclusion that with the CRIT criterion the nominal designs are safer than those of RDF87, for the probabilistic models is not as evident as it is in the deterministic models.

Finally, it is worth mentioning that in order to increase the structural safety of non-linear asymmetric structures, it is necessary to modify the strength distribution of the structural elements. The results obtained in this study show that further torsion design criteria should include as design parameters, the structural eccentricity, the fundamental natural period and the seismic performance factor. At the same time, the new criteria must be simpler and, if possible, must keep the traditional format of the actual torsion design criteria, since the more complicated the design procedure, the less practical it is.

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